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To cite this article: Y J Gao et al 2014 Smart Mater. Struct. 23 095003

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Performance of bistable piezoelectric cantilever vibration energy harvesters with an elastic support external magnet

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Received 19 January 2014, revised 5 June 2014
Accepted for publication 6 June 2014
Published 16 July 2014

Abstract
Based on the research results of conventional rigid support nonlinear energy harvesters, in this paper we conceive a kind of structure with an elastic support external magnet with the intent to keep the system in the state of bistable oscillation, even under low-intensity excitation conditions. It has been proved that elastic support systems have better power output performance than rigid support systems when excited at low-intensity vibrations. In addition, elastic support nonlinear energy harvesters do not need real-time adjustment of the magnet interval towards the variable-intensity random excitation source, consequently achieving maximum power output and sufficient electromechanical energy conversion of the system.

Keywords: energy harvesting, bistable oscillation, piezoelectric cantilever, elastic support

(Some figures may appear in colour only in the online journal)

1. Introduction

Thanks to the recent advances in low-power electronics in the past decades, the conversion of dispersed energy from ambient sources into electrical energy has been revealed to be a realistic alternative for powering wireless sensors and self-powered devices [1–3]. Among various renewable forms of energy, mechanical vibration is deemed to be one of the most attractive power sources for low-power electronics owing to its power density, versatility, and abundance in real environments [4, 5]. The conversion of mechanical energy from background vibrations into electrical energy via a piezoelectric effect is particularly effective, this is a unique property of certain crystals which will generate electrical charge if subjected to mechanical stress or vibration [6, 7]. Piezoelectric energy harvesting has a great advantage of high energy density, simple structure, and it is easy to be miniaturized, which plays a significant role when it comes to the combination of MEMS technology and piezoelectric energy harvesting [8–10].

Conventional linear piezoelectric energy harvesters generate maximum power when their resonance frequency matches the ambient vibration frequency. However, ambient vibration energy is often located below a few hundred hertz with variable intensity and it is distributed over a wide spectrum [11]. The performance of linear energy harvesters will be significantly affected when excited at broadband vibrations. Thus, the narrowband disadvantage restricts the application of linear energy harvesters in many realistic environments. Many methods have been explored by several research groups to broaden the usable bandwidth of linear harvesters, including oscillator arrays, multi-modal oscillators, and active or adaptive frequency-tuning methods [12, 13]. While providing improvements, more advanced solutions were desired for broadband performance, and subsequently the exploitation of nonlinear dynamic phenomena became a focus of this research. Numerous nonlinear piezoelectric energy harvesting approaches have been introduced, including monostable Duffing, impact and bistable oscillator designs [14, 15]. Among those nonlinear devices, bistable energy harvester oscillating between two potential wells has become a popular research topic. Bistable oscillators have a unique double-well restoring force potential, which leads to optimal displacement and electric output. Among many
alternatives, the three most common bistable harvester concepts are composed of magnet repulsion harvesters, magnetic attraction harvesters and buckled beam harvesters [16, 17]. Recent research investigations have found that nonlinear vibration energy harvesters perform better in vibration energy harvesting when compared to linear devices under broadband excitations for a specific excitation intensity range, which is slightly above the threshold of interwell oscillations, while obtaining maximum power output [18–20]. It has been proven that the maximum power output of nonlinear vibration energy harvesters under random excitations are 4–6 times that of linear counterparts, which indicates the superiority of nonlinear vibration energy harvesters [21, 22].

A conventional bistable magnet repulsion harvester, typically coupled with a rigid support external magnet, bounces between two stable states when excited by random vibrations, resulting in electric output generation. Nevertheless, the electric output of these devices is significantly affected by the intensity of the environmental vibrations. When the ambient vibration excitation intensity is insufficient, the piezoelectric cantilever may be limited in either potential well, while generating a weak oscillation, resulting in a lack of bistable transition oscillation and a decline of the power output. In this context, with the aim of overcoming the defects of conventional nonlinear piezoelectric cantilever vibration energy harvesters, we conceive of a kind of structure with an elastic support external magnet with the intent to ensure that the system will oscillate in a bistable state under low-intensity excitation conditions, thus achieving maximum power output and sufficient electromechanical energy conversion of the system.

2. Bistable piezoelectric cantilever vibration energy harvesters with a rigid support external magnet

2.1. Basic structures and system model

Figure 1 shows the basic structures and force analysis of bistable piezoelectric cantilever vibration energy harvesters with a rigid support external magnet (hereinafter referred to as ‘rigid support energy harvesting systems’ or ‘rigid support systems’) [23, 24]. The converter is composed of a piezoelectric cantilever B with a magnet A, which is fixed at its free end, a rigid support external magnet C, and a base D, which is excited by ambient vibrations. With respect to the piezoelectric cantilever B, we use a metal plate as substrate. On both the substrate surfaces, piezoelectric ceramic films (PZT) are deposited to perform the energy conversion. The piezoelectric bimorphs are of the same thickness and connected in series. Magnets A and C are mutually exclusive, forming a bistable system. The principle of the converter is that when excited by ambient sources, the oscillation of piezoelectric cantilever B would lead to deformations of PZT films, thus the conversion of mechanical energy from background vibrations into electrical energy via the piezoelectric effect can be achieved. Note that when the system is at the equilibrium position, the effect of magnet A’s gravity on the static deformation of the piezoelectric cantilever beam is not considered. Meanwhile, magnet A is located along the horizontal extension line of the cantilever while magnets A and C are horizontally aligned.

Figure 1 may be simplified as a typical mass-spring-damper model, its equivalent model is shown in figure 2. According to Newton’s second law, we receive dynamic equations of the system as follows [23]:

\[ kP(t) + \dot{V}(t) + F_i = M_{eq}\ddot{Z}(t) + \eta_{eq}\dot{Z}(t) + K_{eq}Z(t) \]  

where \( M_{eq}, K_{eq} \) and \( \eta_{eq} \) respectively, represent equivalent mass, equivalent stiffness, and equivalent damping of the piezoelectric cantilever part; \( k \) is electromechanical coupling coefficient of piezoelectric ceramics (PZT); \( k \) is the amplitude correction factor of a lumped parameter model; \( P(t) \) represents ambient vibrations; \( F_i \) is the vertical component of magnetic repulsive force \( F \); \( Z(t) \) represents the displacement of \( M_{eq} \); and, \( V(t) \) is the output voltage of PZT.

\[ M_{eq} = M + \frac{33m}{140} \]  

\[ K_{eq} = \frac{6E_I I}{l_0^2 (2l_b + 1.5l_w)} \]  

\[ \eta_{eq} = 2M_{eq}\xi_{eq}\omega_r \]  

\[ \theta = \frac{e_{31} e_{33} g^{2} (l_w + l_b)}{2} \]  

In formula (2), \( M \) represents tip mass (i.e. mass of magnet A), with a calculation formula of \( M = \rho_b l_b w_b h_b \), and
m represents mass of the piezoelectric cantilever, with a calculation formula of \( m = \rho_l l_b w_b t_b + 2\rho_l l_s w_s t_s \). In formula (3), \( I \) is the rotational inertia with a calculation formula of
\[
I = 2 \left( \frac{\omega_{33} S^2}{12} + w_s l_s \left( \frac{I_b + I_s}{2} \right) \right) + \frac{\omega_{31} w_b t_b}{12} \]  
[27]. \( E_b \) is the elasticity modulus of cantilever substrate, and \( E_s \) is the elasticity modulus of piezoelectric ceramic. In formula (4), \( \omega_s \) is the structural natural frequency of the piezoelectric cantilever part, with a calculation formula of \( \omega_s = \sqrt{K_{ij}/M_{ii}} \), and \( \xi \) is the mechanical damping ratio. In formula (5), \( \epsilon_{33} \) is the piezoelectric constant, and \( \psi \) is the spatial derivative of the mechanical mode shape. In addition to the above, the other parameters used in formulas (2)–(5) are described as follows: \( \rho_b, l_b, w_b, \) and \( h_b \), respectively, represent the density, thickness, width, and height of either magnet, as shown in figure 3; \( \rho_s, l_s, w_s, \) and \( t_s \), respectively, represent the density, length, width, and thickness of the cantilever substrate; \( \rho_{m}, l_{m}, w_{m}, \) and \( t_{m} \), respectively, represent the density, length, width, and thickness of the piezoelectric ceramic.

Studies have shown that, for horizontal or vertical cantilever vibrations, since a traditional lumped parameter vibration response would produce an error, we need to introduce a correction factor for excitation amplitude correction so that the revised lumped parameter model agrees well with the distributed parameter model within a wide range nearby first-order vibration frequency to meet the accuracy requirements of the dynamic simulation. The mathematical expression of the correction factor \( k \) is [25]:
\[
k = \frac{(M/m)^2 + 0.603(M/m) + 0.08955}{(M/m)^2 + 0.4637(M/m) + 0.05718}. \]  
(6)

According to Kirchhoff’s first law, we receive the circuit equation of acquisition circuit [28]:
\[
\theta Z(t) + \frac{1}{2} C_r \ddot{V}(t) + \frac{V(t)}{R_L} = 0 \]  
(7)
where \( R_L \) is the resistive load and \( C_r \) is the coupling capacitance, which has a calculation formula of [28]:
\[
C_r = \frac{\epsilon_{33}^S w_b l_b}{t_s} \]  
(8)
where \( \epsilon_{33}^S \) represents dielectric constant of medium. \( \epsilon_{33}^S \) is given by \( \epsilon_{33}^S = \epsilon_{33} \epsilon_0 \), where \( \epsilon_{33} \) represents relative permittivity and \( \epsilon_0 \) represents vacuum permittivity.

2.2. Analysis of nonlinear potential function

According to the calculation methods for the magnetic force, at the equilibrium position repulsive force \( F \) is [23]:
\[
F = \frac{1.5}{1+3d} \times \frac{w_b h_b}{2\mu_0} \times \left[ \tan^{-1} \frac{w_b h_b}{2d \sqrt{w_b^2 + h_b^2 + 4d^2}} - \tan^{-1} \frac{w_b h_b}{2d \sqrt{w_b^2 + h_b^2 + 4d^2}} \right] \]  
(9)
where \( d \) is the interval between two magnets, as shown in figure 3, \( \mu_0 \) is the permeability of the vacuum, and \( B_i \) is one of the magnetic parameters of the permanent magnets.

The vertical component of magnetic repulsive force \( F_v \) varies with the vertical displacement of magnet \( AZ(t) \), as shown in figure 1, which is:
\[
F_v = F \times \frac{Z(t)}{\sqrt{Z(t)^2 + d^2}}. \]  
(10)

Without consideration of gravity, the potential energy of the system includes the elastic potential energy of the equivalent model and work done on the magnets by \( F \), from which we can obtain the potential function of a rigid support energy harvesting system at the time of \( Z = Z_0 \):
\[
V(Z_0) = \int_0^{Z_0} K_0 Z dZ - \int_0^{Z_0} \sqrt{Z_0^2 + d^2 - d} F dZ. \]  
(11)

By integration we can obtain the trend of potential function \( V(Z) \) with respect to magnet interval \( d \), as shown in figure 4. As we can see, when magnet interval \( d \) is large, especially when \( d \) is infinite, the system is nearly linear with a monostable potential function, which has only one stable...
equilibrium position near the origin. With the decrease of interval \( d \), the system becomes nonlinear with a bistable potential function, which has two stable equilibrium positions in two wells, as well as one unstable equilibrium position near the origin.

2.3. Numerical simulation analysis

In order to simulate the performance of rigid support energy harvesting systems excited by ambient vibrations, according to references [26–29], we select a set of fixed parameters, including material properties (shown in Table 1) and geometric dimensions (shown in Table 2), while studying the response of the system’s output displacement and output voltage by numerical calculation. In addition, the other main parameters required for simulation analysis are: \( \xi = 0.0178 \) and \( R_L = 10 \, \text{M} \Omega \).

In this paper, random excitation with intensity of 1 and frequency bandwidth of 0~120 Hz is utilized as vibration input to simulate low-frequency noise sources in the environment. The system’s vibration response under random excitations, that is displacement of magnet \( AZ(t) \) and output voltage of piezoelectric beams \( V(t) \) can be stimulated according to (1) and (7) by the Runge–Kutta method [30–32]. The rms output voltage \( V_{\text{rms}} \) over the considered bandwidth is a suitable indicator of the power deliverable to the purely resistive load \( R_L \) [20, 24]. The average electrical power produced by the piezoelectric oscillator is calculated by the formula \( P_{\text{avg}} = V_{\text{rms}}^2/R_L \) [21, 22]. Numerical simulations indicate that \( V_{\text{rms}} \) increases at first and then decreases as \( d \) increases, while maintaining a greater level of peak around the interval of 4.2 mm. Phase portraits for three interval positions (\( d < 4.2 \, \text{mm} \), \( d = 4.2 \, \text{mm} \) and \( d > 4.2 \, \text{mm} \)) are, respectively, shown in figures 5(a)–(c). [33] By combining figures 4 and 5 we can see that for \( d < 4.2 \, \text{mm} \), although the potential function is bistable, the transition oscillation hardly occurs between two wells because of large potential wells and a high barrier, thus resulting in small oscillation in either well with little output displacement and voltage. For \( d = 4.2 \, \text{mm} \), the barrier of the bistable function potential decreases so that reciprocating transition oscillation could occur between two wells, from which output displacement and voltage are characterized by a wide margin of alternatively positive and negative variations while the rms output voltage \( V_{\text{rms}} \) is optimal. For \( d > 4.2 \, \text{mm} \), the system becomes nearly linear with a monostable potential function, output displacement, and voltage decrease along with linear oscillation.

Additionally, for spectral comparison, figure 6 reports the response for linear and nonlinear status (\( d = 4.2 \, \text{mm} \)) in the frequency domain, which is represented as velocity power spectral density (PSD) [20]. As is shown, the nonlinear bistable case provides a spectrum with a wider bandwidth compared to resonant behavior of the linear case. The PSD of

<p>| Table 1. Material properties used for rigid support energy harvesters. |
|---------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever substrate material properties: Brass</td>
<td></td>
</tr>
<tr>
<td>( E_b ) (GPa)</td>
<td>100</td>
</tr>
<tr>
<td>( \rho_b ) (kg m(^{-3}))</td>
<td>7165</td>
</tr>
<tr>
<td>Piezoelectric ceramic material properties: PZT-5A</td>
<td></td>
</tr>
<tr>
<td>( E_e ) (GPa)</td>
<td>66</td>
</tr>
<tr>
<td>( \rho_e ) (kg m(^{-3}))</td>
<td>7800</td>
</tr>
<tr>
<td>( \varepsilon_{31} ) (pF m(^{-1}))</td>
<td>1500</td>
</tr>
<tr>
<td>( \varepsilon_{33} ) (pF m(^{-1}))</td>
<td>8.854</td>
</tr>
<tr>
<td>( d_{31} ) (pC/N)</td>
<td>−190</td>
</tr>
<tr>
<td>Magnet material properties: Nd(<em>2)Fe(</em>{14})B</td>
<td></td>
</tr>
<tr>
<td>( \rho_m ) (kg m(^{-3}))</td>
<td>7500</td>
</tr>
<tr>
<td>( B_r ) (T)</td>
<td>1.25</td>
</tr>
<tr>
<td>( \mu_0 ) (N/A(^2))</td>
<td>( 4\pi \times 10^{-7} )</td>
</tr>
</tbody>
</table>

<p>| Table 2. Geometric dimensions used for rigid support energy harvesters. |
|---------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_d, l_e )</td>
<td>64</td>
</tr>
<tr>
<td>( w_b, w_e )</td>
<td>10</td>
</tr>
<tr>
<td>( t_e )</td>
<td>0.27</td>
</tr>
<tr>
<td>( t_b )</td>
<td>0.14</td>
</tr>
<tr>
<td>( l_B )</td>
<td>8</td>
</tr>
<tr>
<td>( h_B )</td>
<td>20</td>
</tr>
<tr>
<td>( w_B )</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 5. Phase portraits for \( D = 1 \) when (a) \( d < 4.2 \, \text{mm} \), (b) \( d = 4.2 \, \text{mm} \), (c) \( d > 4.2 \, \text{mm} \).
the bistable mass velocity is greater than that for the linear sample, except at the linear device natural frequency. This is expected to produce an improved effectiveness in converting wide-spectrum vibrations.

In view of the $d = 4.2$ mm case, which presents high rms output voltage $V_{rms}$, in this paper we define $D = 1$ to represent high-intensity random excitation conditions while $D = 0.1$ is used to represent the low-intensity random excitation conditions. Note that the definition of the density (high or low) locates in the transition oscillation rate during a period of time. When the rate is higher than 50%, in this paper we assume that the system is under high-intensity random excitation conditions. Otherwise, when the rate is relatively low or close to zero, that is transition oscillation barely happens, we stipulate that the system is under low-intensity circumstances. In terms of the case when excitation intensity $D$ decreases to 0.1, output displacement $Z(t)$ and output voltage $V(t)$ are measured by the Runge–Kutta method. In order to contrast the response of the device under two kinds of excitation intensity, we draw four comparison diagrams concerning output displacement, output voltage, phase portraits and frequency spectra of output voltage, as respectively shown in figures 7(a)–(d). As is shown, when random excitation intensity is insufficient, small oscillation occurs in either well instead of transition oscillation between two wells, thereby decreasing the power output of the system.

The simulation results above demonstrate that for nonlinear cantilever beam structures with certain magnet interval, reciprocating transition oscillation between two wells only happens when the vibration excitation intensity is sufficient. In this situation, the maximum output voltage of energy harvesting can be obtained. Nevertheless, when vibration excitation intensity is insufficient, it is apparent that this kind of rigid support energy harvesting system is not capable of realizing reciprocating transition oscillation between two wells. The response of a piezoelectric cantilever is limited in either well generating small oscillation, which thereby decreases the power output of the system. In view of this, in this paper we conceive of a kind of structure with an elastic support external magnet to study the performance of nonlinear piezoelectric cantilever vibration energy harvesting under low-intensity excitation conditions.

3. Bistable piezoelectric cantilever vibration energy harvesters with an elastic support external magnet

3.1. Basic structures and the system model

The basic structures and force analysis of bistable piezoelectric cantilever vibration energy harvesters with an elastic support external magnet (hereinafter referred to as ‘elastic support energy harvesting systems’ or ‘elastic support systems’) are shown in figure 8 [34]. The difference between this structure and a conventional rigid support structure would be the fact that magnet $C$ is elastic supported, which is realized by a cantilever structure to ensure maximum vertical motion of magnet $C$ (detailed information is provided in section 5). Here we assume only vertical movements of magnet $C$ without considering swing motions in the horizontal direction. This improved structure not only retains vibration bistability of the cantilever but also introduces bistability into an external magnet’s vibration characteristics, which consequently create an opportunity for the reciprocating transition oscillation of a cantilever under low-intensity excitation conditions. Note that when the system is at the equilibrium position, the effect of the gravity of magnets $A$ and $C$ on the static deformation of the piezoelectric cantilever beam and the spring is not considered, meanwhile magnet $A$ is located along the horizontal extension line of the cantilever while magnets $A$ and $C$ are horizontally aligned.

Figure 8 can also be simplified as a typical mass-spring-damper model. Its equivalent model is shown in figure 9. According to Newton’s second law, we receive the dynamic equations of the system as follows:

$$kP(t) + \dot{\theta}V(t) + F_i = M_{eq} \ddot{Z}(t) + \eta \dot{Z}(t) + K_{eq}Z(t) \quad (12a)$$

$$kP(t) - F_i = M_{eq} \dot{Z}(t) + \eta \dot{Z}(t) + K_{eq}Z(t). \quad (12b)$$

Similar to (1), in (12a), $M_{eq}$, $K_{eq}$, and $\eta$, respectively, represent equivalent mass, equivalent stiffness, and equivalent damping of the piezoelectric cantilever part; $\theta$ is electromechanical coupling coefficient of piezoelectric ceramics (PZT); $k$ is the amplitude correction factor of a lumped parameter model; $P(t)$ represents ambient vibrations; $F_i$ is the vertical component of magnetic repulsive force; $Z(t)$ represents the displacement of $M_{eq}$; and, $V(t)$ is the output voltage of the PZT. In (12b), $M_i$ represents mass of external magnet $C$, which has the same quality with magnet $A$, $Z_i(t)$ is the displacement of $M_i$, $K_i$, and $\eta_i$, respectively, represent stiffness and damping of the spring. Similar to (4), $\eta_i$ can be defined as follows:

$$\eta_i = 2M_i\xi_ao_i \quad (13)$$

where $\alpha_i$ is the structural natural frequency of the elastic support part, with a calculation formula of $\alpha_i = \sqrt{K_i/M_i}$, and $\xi_0$ is the mechanical damping ratio. The spring’s elastic
stiffness $K_n$, which is used as one of the key parameters to define the elastic support state, will be discussed later in this paper. The circuit equation of acquisition circuit of elastic support systems is the same as (7).

3.2. Analysis of nonlinear potential function

According to the calculation methods for magnetic force, repulsive force $F$ is also defined by (9). According to the force analysis in figure 8, the vertical component of magnetic repulsive force $F$ (i.e. $F_v$) is different from that in (10). Due to the existence of vibration displacement of magnet $CZ_n(t)$, $F_v$ can be defined as follows:

$$F_v = F \times \frac{Z(t) - Z_n(t)}{\sqrt{[Z(t) - Z_n(t)]^2 + d^2}}. \quad (14)$$

Without consideration of gravity, potential energy of the elastic support energy harvesting system can be divided into two parts. One is potential energy gained by vibrations of the cantilever $V(Z)$, while the other one forms by vibrations of external magnet $CV(Z_n)$. Similar to the derivation of (11), it is easy to define $V(Z)$ at the time of $Z = Z_n$, as well as $V(Z_n)$ at
the time of $Z_n = Z_{n_0}$:

$$V(Z_0) = \int_0^{Z_0} K_{eq} Z dZ - \int_0^{Z_t} \sqrt{Z_0^2 + d_1^2} - d_1 F dZ$$ (15a)

$$V(Z_{n_0}) = \int_0^{Z_{n_0}} K_{eq} Z_n dZ_n - \int_0^{Z_t} \sqrt{Z_{n_0}^2 + d_2^2} - d_2 F dZ_{n_0}$$ (15b)

When magnets A and C move to any positions, $d_1$ and $d_2$, respectively, represent the horizontal spacing between each center of gravity and the crossing point of the connecting line of their centers of gravity and the horizontal extension line of the cantilever, as shown in figure 8, while meeting the condition of $d_1 + d_2 = d$. Meanwhile, $d$ still represents the interval between magnets A and C. Apparently, the potential function of the elastic support system in (15) is identical to that (11) in the form, which suggests that elastic support systems also have bistability.

As seen in figure 8, $d_1$ and $d_2$ meet the condition of $d_1/d_2 = Z(t)/Z_n(t)$, which means that for a fixed spacing value of $d_1$, $d_2$ and $d$ could take any value within the interval of $[0, d]$ at any moment in the vibration process of the two magnets. Using the parameters in tables 1 and 2 while supposing $d = 4.2 \text{ mm}$, $K_e = 0.6K_{eq}$, five different values of $d_1$ ($d/100$, $d/14$, $d/2$, $3d/4$, $d$) are given to receive the trends of $V(Z)$ and $V(Z_n)$ with respect to certain $d_1$ in the elastic support systems, as shown in figures 10(a) and (b). Obviously, bistable potential functions determine whether the cantilever or the external magnet has variable shapes of curves along with $d_1$.

As shown in figure 10, the potential function of elastic support systems when $d_1 = d$ coincides with that of the rigid support systems. At this point, the cantilever vibrates to either of the potential wells that have deviated from the equilibrium position. When $d_1 = 0$, the cantilever is at the equilibrium position. It is also apparent that when $d_1$ takes any value within the interval of $[0, d]$ then the potential functions of elastic support systems $V(Z)$ and $V(Z_n)$ will be different; that is, under random excitation, the potential function of the cantilever will vary randomly with $d_1$ at different moments, which is reflected in the random variation of the barrier, well spacing and shape of the potential function. Hence, we assume that when the cantilever (or external magnet) vibrates in a certain deeper well with a large potential barrier then the external magnet (or cantilever) would be in a more shallow well with a small potential barrier. Once the external magnet (or cantilever) happens to transit from one well to another then the cantilever (or external magnet) would transit along, which creates a corresponding increase of the transition probability and thus enhances the transition frequency. A frequent bistable transition generated by piezoelectric beams would consequently help achieve maximum power output and sufficient electromechanical energy conversion of the system.

### 3.3. Numerical simulation analysis

Assuming that excitation intensity is $D = 1$, then by adjusting magnet interval $d$ and the elastic stiffness of the spring $K_e$, the displacement of magnet $AZ(t)$ and output voltage of piezoelectric beams $V(t)$ can be stimulated according to (12) and (7) by the Runge–Kutta method. We can find that a frequent bistable transition oscillation of elastic support systems occurs when $d = 8 \text{ mm}$, $K_e = 0.6K_{eq}$ around the equilibrium position with a maximum value of rms output voltage $V_{rms}$. In order to discuss the different responses for rigid and elastic support systems around the position of $d = 8 \text{ mm}$, the comparison diagrams shown in figures 11(a)–(d), respectively, represent the output displacement, output voltage, phase portraits, and frequency spectra of the output voltage. Similar to but unlike nearly linear oscillation in rigid support systems, figure 11 presents frequent bistable transition in elastic support systems, whose displacement and output voltage are obviously greater than that in nearly linear systems. The effect of dynamic interval $d_1$ on potential function $V(Z)$ in elastic support systems when $d = 8 \text{ mm}$ is shown in figure 12. As is shown, the potential function of elastic the support system varies randomly with $d_1$ at different moments. Note that the potential function in figure 12, when $d_1 = d$, coincides with that in figure 4. Hence,
we notice that in contrast from the potential function in rigid support systems, which remains nearly linear, the variable potential function in elastic support systems provides an opportunity for bistable transition oscillation.

Furthermore, in order to contrast the response for rigid and elastic support systems under low-intensity random excitation conditions, we obtain another group of output displacements, output voltages, phase portraits and frequency spectra of output voltage, as respectively shown in figures 13(a)–(d). Obvious as it is, when random excitation intensity is insufficient \((D = 0.1)\), compared to the incapability of bistable transition oscillation in rigid support systems but small oscillation in one well systems (also shown in figure 7), we find that a frequent bistable transition oscillation occurs in elastic support systems around the equilibrium position. Thanks to the system’s variable potential function, it is more likely for bistable transition oscillation to occur. It is also demonstrated that the elastic support system is capable of adapting to random excitations with variable intensity, through which maximum power output and sufficient electromechanical energy conversion of the system can be accomplished.

### 4. Comparative analysis of two kinds of energy harvesters

For ease of comparison, here we use the ratio of the spring’s elastic stiffness \(K_s\) to the cantilever’s equivalent stiffness \(K_{eq}\) to reflect the support state of the systems. The greater that \(K_s/K_{eq}\)
is, the closer the external magnet \( C \) is to the rigid support state. When \( K_s/K_{eq} \) becomes infinite, the entire system can be seen as rigidly supported. On the contrary, the less that \( K_s/K_{eq} \) is, the closer the system is to the elastic support state.

### 4.1. Effect of magnet interval and elastic stiffness on energy harvesting performance

Firstly, we observe the variation of rms output voltage \( V_{rms} \) with respect to magnet interval \( d \) and the spring’s elastic stiffness \( K_s \) under high-intensity random excitation conditions. According to the analysis above, supposing that \( D = D_1 \) represents high-intensity level of random excitations, which remain unchanged in this part of discussion, we can gain rms output voltage \( V_{rms} \) by statistical averaging of the output voltage \( V(t) \) within a period of time. The dependence of \( V_{rms} \) on \( d \) and \( K_s/K_{eq} \) is shown in figure 14(a), suggesting that: 1) When \( K_s/K_{eq} \) is within the interval of \([0, 1]\) and \([2, 10]\), \( V_{rms} \) increases at first and then decreases as the interval \( d \) increases. Notably, within the small stiffness interval of \([0, 1]\), \( V_{rms} \) maintains a greater level of peak with the interval in the range of \( 5.8 \sim 9.0 \) mm, while within the large stiffness interval \([2, 10]\), \( V_{rms} \) maintains a greater level of peak only around the interval of \( 4.2 \) mm. 2) For any interval \( d \), \( V_{rms} \) increases at first and then decreases as \( K_s/K_{eq} \) increases, a maximum value and a minimum value occurring at, respectively, \( K_s = 0.6K_{eq} \) and \( K_s = K_{eq} \). Then, \( V_{rms} \) continues to increase along with the increase of \( K_s/K_{eq} \), eventually tending to a constant value. Apparently, in elastic support systems, the greater the spring’s elastic stiffness \( K_s \) is (i.e. the closer the system is near rigid support state), the smaller interval is needed for a larger output voltage. On the contrary, when \( K_s \) is smaller, the system would perform better with a larger interval. This indicates that under high-intensity excitation conditions, like \( D = 1 \), for rigid or nearly rigid support systems, frequent bistable transitions occur around smaller magnet intervals, and its optimal structure parameter should be \( d = 4.2 \) mm. However, for elastic support systems, frequent bistable transitions occur around larger magnet intervals, and its optimal structure parameters should be \( d = 8 \) mm as well as \( K_s = 0.6K_{eq} \).

Secondly, we observe the variation of rms output voltage \( V_{rms} \) with respect to magnet interval \( d \) and the spring’s elastic stiffness \( K_s \) under low-intensity random excitation conditions. Supposing that \( D = 0.1 \) represents the low-intensity level of random excitations, which remain unchanged in this part of discussion, we can similarly gain the dependence of \( V_{rms} \) on \( d \) and \( K_s/K_{eq} \), as shown in figure 14(b). Note that figures 14(a) and (b) are very similar in shape. Notably, under low-intensity excitation conditions, the optimal structure parameters point of rigid support systems is no longer at the position of \( d = 4.2 \) mm but transfers to the position of \( d = 5.8 \) mm.

![Figure 13.](image-url)

Figure 13. (a) Output displacement, (b) output voltage, (c) phase portraits, (d) frequency spectra of output voltage for \( D = 0.1 \) when \( d = 8 \) mm, \( K_s = 0.6K_{eq} \) in elastic support systems and \( d = 4.2 \) mm in rigid support systems.
Nevertheless, the optimal structure parameters point of elastic support systems is still around the position of $d = 8\text{mm}$, $K_n = 0.6K_{eq}$.

A comparison of figures 14(a) and (b) demonstrates that under high-intensity excitation conditions, like $D = 1$, the maximum rms output voltage of rigid or nearly rigid support systems at the position of $d = 4.2\text{mm}$, which is $V_{rms} = 0.9719$, is higher than that of elastic support systems at the position of $d = 8\text{mm}$, which is $V_{rms} = 0.9042$. Note that in figure 14, $d$, $K_n/K_{eq}$ and $V_{rms}$ correspond, respectively, to $X$ axis, $Y$ axis and $Z$ axis. However, under low-intensity excitation conditions, like $D = 0.1$, the maximum rms output voltage of rigid or nearly rigid support systems at the position of $d = 4.2\text{mm}$, which is $V_{rms} = 0.2066$, whose maximum has been transferred to the position of $d = 5.8\text{mm}$ with the value of $V_{rms} = 0.4215$, is lower than that of elastic support systems at the position of $d = 8\text{mm}$, which is $V_{rms} = 0.2988$. Since in practical applications the structure parameters of energy harvesters cannot be changed in real time, if considered remaining unchanged magnet interval and stable maximum voltage output, elastic support systems have more superiority than rigid support systems.

4.2. The effect of magnet interval and excitation intensity on energy harvesting performance

In order to compare the characteristics of rigid support systems and elastic support systems, we take $K_n = 10K_{eq}$ and $K_n = 0.6K_{eq}$ to respectively represent them, which remain unchanged in this part of discussion. The dependence of rms output voltage $V_{rms}$ of these two systems on excitation intensity $D$ and magnet interval $d$ is shown in figure 15. As can be seen, $V_{rms}$ increases at first and then decreases as $d$ increases while it increases along with the increase of $D$. Interestingly, for rigid support systems (shown in figure 15(a)), when the excitation intensity $D$ decreases, the position of interval $d$ where the maximum value of $V_{rms}$ occurs varies with it, while for elastic support systems (shown in figure 15(b)) it remains the same, which proves some of the demonstrations above. Considering that the remaining structural parameters are unchanged, we choose a profile of figure 15(a) at the point of $d = 4.2\text{mm}$ and a profile of figure 15(b) at the point of $d = 8\text{mm}$ to observe the dependence of $V_{rms}$ on $D$ in the two systems, as shown in figure 16. Note that under low-intensity random excitation conditions, elastic support systems have better power output performance than rigid support systems. In contrast, when it comes to high-intensity random excitation conditions, rigid support systems play a better role.
5. Experimental analysis

To verify our theoretical analysis and simulation results, we manufactured the piezoelectric cantilever energy harvesting structures using the parameters in table 1 and table 2 for experimental analysis. The type of piezoelectric bimorph is PZT-5A. Two layers of piezoelectric ceramics are deposited in the same direction of polarization while they are closely adhered to the intermediate electrode layer. Wires settled on the PZT surfaces are used for voltage output. The material of the intermediate electrode layer (i.e. the cantilever substrate) is brass. The piezoelectric cantilever has a permanent magnet $A$, whose type is N35, fixed to the free end while its root is fixed to base 1. Meanwhile, base 1 is fixed to the bottom plate of the energy harvesting structures (i.e. base 2), as shown in figure 17(a). In order to facilitate the comparative experiment of elastic support systems and rigid support systems, we use a silicon steel cantilever (thickness of 0.25 mm) to play the same role as the spring structure in equivalent models when designing physical models of elastic support systems, as shown in figure 17(a). The silicon steel cantilever has permanent magnet $C$ fixed to the free end while its root is fixed to base 3. Meanwhile, base 3 is capable of moving horizontally along base 2 and along the length direction of the cantilever beam for adjustment of the interval between the two magnets. Both the planes of piezoelectric cantilever beam and silicon steel cantilever are parallel to that of base 2, through which the entire energy converter receives excitations from the exciter.

Note that adjustment of length of silicon steel cantilever $l_k$ within a range of $[0, 40 \text{ mm}]$ equals to adjustment of external magnet $C$’s elastic support state. In other words, according to the fact that elastic force $F_x$’s elastic deformation $X_x$ and the spring’s elastic stiffness $K_x$ meet the relation of $F_x = -K_xX_x$ as well as the fact that $l_k$ is proportional to $X_x$, the variation of $K_x$ could be simulated by the variation of $l_k$. The less that $l_k$ is, the closer the external magnet $C$ is to the rigid support state. When $l_k = 0$, the entire system can be seen as rigidly supported. In contrast, the greater that $l_k$ is, the closer the system is to the elastic support state. The purpose of designing the silicon steel cantilever structure in the experiment can be demonstrated as follows: 1) Adjustment of spring’s elastic stiffness $K_n$ can be achieved merely by adjusting the length of silicon steel cantilever $l_k$, guaranteeing the consistency of other parameters when conducting comparative experiments. 2) The system structures shown in figure 17(a) meet the assumption mentioned in 3.1 of ensuring the maximum vertical motion of magnet $C$. Furthermore, in order to satisfy the assumption mentioned in 3.1 of not considering the effect of the gravity of magnets $A$ and $C$ on the static deformation of the piezoelectric cantilever beam and spring when the system is at the equilibrium position, in the experiment the planes of the two beams and base 2 are deposited perpendicularly to the ground, as shown in 16 (b). In this situation, the exciter vibrates base 2 along a direction parallel to the ground so as to eliminate the effect of the magnets’ gravity on the static deformation of the piezoelectric cantilever beam and spring.

The experimental test system diagram is shown in figure 18, which is mainly composed of a signal generator, power amplifier, exciter, piezoelectric cantilever energy...
harvester, data acquisition system (DAQ), computer, etc [28, 35]. In the experiment, the signal generator can produce random excitations with a frequency bandwidth of $0 \sim 120$ Hz and different intensities. Random excitation signals act on the energy converter through the power amplifier, leading to vibrations of the piezoelectric cantilever. Voltage generated over a resistive load can be acquired via wires and DAQ, and then put into a computer for analysis.

By adjusting magnet interval $d$ and length of silicon steel cantilever $l_k$, in the experiment we obtain a set of mean square values of output voltage $V_{\text{rms}}$, respectively, under random excitation conditions with high-intensity of $D = 1.5$ and a low-intensity of $D = 0.5$ when $d$ and $l_k$ take different values. In accordance with figures 14(a) and (b), we can gain the dependence of $V_{\text{rms}}$ on $d$ and $l_k$, as shown in figures 19(a) and (b). As we can see, whether under random excitation of high-intensity or low-intensity, figures 19(a) and (b) are extremely similar with figures 14(a) and (b) in terms of their shapes, indicating that voltage output characteristics and variation tendency in both rigid support systems and elastic support systems coincide with the simulation results, consequently verifying the theoretical analysis.

Further observation on figure 19 shows that no matter under random excitation conditions with high-intensity of $D = 1.5$ or low-intensity $D = 0.5$, in elastic support systems $V_{\text{rms}}$ maintains a greater level of peak within magnet interval of $35 \sim 45$ mm around the position of $l_k = 30$ mm. For rigid support systems (when $l_k = 0$), when the excitation intensity $D$ decreases, the maximum value of $V_{\text{rms}}$ varies with it. For further observations, in accordance with figure 15, by adjusting magnet interval $d$ and excitation intensity $D$, in the experiment we obtain a set of mean square values of output voltage $V_{\text{rms}}$, respectively, under conditions with $l_k = 30$ mm and $l_k = 0$ when $d$ and $D$ take different values, as shown in figure 20. Notably, for the rigid support systems shown in figure 20(a), when the excitation intensity $D$ decreases, the position of interval $d$ where the maximum value of $V_{\text{rms}}$ occurs transfers from $30$ mm to $25$ mm while for elastic support systems, as shown in figure 20(b), it remains the same at around $35$ mm. Since in practical applications the structure parameters of energy harvesters cannot be changed in real time, if considered remaining unchanged magnet interval and stable maximum voltage output, elastic support systems have more superiority than rigid support systems, consequently verifying the conclusions in section 4.

Considering that the remaining structural parameters are unchanged, we choose a profile of figure 20(a) at the point of $d = 30$ mm and a profile of figure 20(b) at the point of $d = 35$ mm to observe the dependence of $V_{\text{rms}}$ on $D$ in the two systems, as shown in figure 21. Similar to figure 16, it is
suggested that under low-intensity random excitation conditions the elastic support systems will have a better power output performance than the rigid support systems. In contrast, the rigid support systems play a better role for the high-intensity random excitation conditions, further verifying the conclusions in section 4.

6. Conclusion

When vibration excitation intensity is insufficient, it is apparent that a rigid support energy harvesting system is not capable of realizing reciprocating transition oscillation between two wells. The response of a piezoelectric cantilever is limited in either well generating small oscillation, which thereby decreases the power output of the system. In view of this defect, in this paper we conceive of a kind of structure with an elastic support external magnet to study the performance of nonlinear piezoelectric cantilever vibration energy harvesting under low-intensity excitation conditions. The system’s variable potential function helps create an opportunity for the reciprocating transition oscillation of a cantilever under low-intensity excitation conditions, thus enhancing the transition probability. It has been proven that under low-intensity random excitation conditions, elastic support systems have better power output performance than rigid support systems. Furthermore, elastic support nonlinear energy harvesting systems do not need the real-time adjustment of magnet interval toward intensity changeable random excitation process that is required by a rigid support system. Finally, it is demonstrated that, when the structural parameters remained unchanged, the elastic support system is capable of adapting to random excitations with variable intensity, through which maximum power output and sufficient electromechanical energy conversion of the system can be accomplished.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No.51275336) and the Specialized Research Fund for the Doctoral Program of High Education of China (Grant No.20120032110001).

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